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ON OBSERVATION PROBLEMS IN DISCRETE SYSTEMS*

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Questions on the construction of information domains consistent with the signal realized at the output of a linear discrete control system with indeterminate noise is examined. Recurrence equations describing the dynamics of the information domains as a function of time and of the signal realized are obtained on the basis of duality relations /1—5/ and of the idea of dynamic programming /6/. Filter equations are obtained from these equations in elementary fashion in the case of consistent quadratic constraints on the perturbations in the system. Certain possibilities of such an approach when considering systems nonlinear in the observations are demonstrated.

1. Statement of the problem and duality theorems. Let an object's motion be described by the difference equation

$$z_{k+1} = Az_k + v_k, \quad k = 0, 1, \ldots, t-1 \tag{1.1}$$

Here z_k is an n-dimensional vector, A is a constant $(n \times n)$-matrix, v_k is a perturbation in the system. It is assumed that it is impossible to observe the vectors z_k and that the quantities

$$y_k = Bz_k + \xi_k \tag{1.2}$$

are measured, where y_k is an m-dimensional vector, B is a constant $(m \times n)$-matrix, ξ_k is a perturbation in the measurement channel. The initial condition z_0 and the perturbations v_k, ξ_k, $k = 0, 1, \ldots, t-1$, are unknown. Information on their possible realizations is exhausted by the descrition of their admissible domains of variation, i.e., by the inclusion

$$\{z_0, \bar{v}_t, \bar{\xi}_t\} \in M \tag{1.3}$$

where M is a closed convex set in $R^{n(t+1)} \times R^{mt}$ and the notation $\bar{f}_t = \{f_0, f_1, \ldots, f_{t-1}\}$ is used.

Definition /1/. The set of those and only those vectors $z \in R^n$ for each of which we can find a triple $\{z_0^*, \bar{v}_t^*, \bar{\xi}_t^*\} \in M$ such that the solution $\bar{y}_t$ of system (1.1), (1.2), found for $z_0 = z_0^*$, $\bar{v}_t = \bar{v}_t^*$, $\bar{\xi}_t = \bar{\xi}_t^*$, $z_t = z$, satisfies the condition $\bar{y}_t = \bar{y}_t^*$, is called a domain $Z_t(\bar{y}_t^*)$ consistent with signal $\bar{y}_t^*$.

From the definition it follows that set $Z_t(\bar{y}_t^*)$ is nonempty and that better information on the true value of vector z_t, than the description of $Z_t(\bar{y}_t^*)$, cannot be obtained. From the linearity of system (1.1), (1.2) and the convexity of M directly follows

Lemma 1.1. Set $Z_t(\bar{y}_t^*)$ is convex.

Set $Z_t(\bar{y}_t)$ is determined uniquely (to within closure) by its own support function

$$W_t(\psi \mid \bar{y}_t) = \sup\{\langle\psi, z\rangle : z \in Z_t(\bar{y}_t)\}$$

Consequently, the determination of the support function of the domain consistent with the realized signal $\bar{y}_t$ consists in solving the following optimal control problem with phase constraints:

$$\langle\psi, z_t\rangle \to \sup \tag{1.4}$$

under conditions (1.1)—(1.3). By writing out the Lagrangian of this problem, we get that the dual problem is described in the form

$$\sup\left\{\langle\psi_0, z_0\rangle + \sum_{k=0}^{t-1}(\langle\psi_{k+1}, v_k\rangle + \langle\eta_k, \xi_k\rangle - \langle\eta_k, y_k\rangle) : \{z_0, \bar{v}_t, \bar{\xi}_t\} \in M\right\} \to \inf, \quad \psi_k = \psi_{k+1}A + \eta_k B, \quad \psi_t = \psi \tag{1.5}$$

Here for simplicity it is assumed that ψ_k and η_k $(k = 0, 1, \ldots, t-1)$ are n- and m-rows, respectively. The following duality theorem is valid under the assumptions made /5/.

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Theorem 1.1. The value of the upper bound in problem (1.4), (1.1)–(1.3) equals the lower bound in the dual problem (1.5).

We can consider the problem for systems with lag, when (1.1) has the form

$$z_{k+1} = Az_k + Cz_{k-l} + v_k,\ k = 0, 1, \ldots, t-1 \tag{1.6}$$

and constraints (1.3) are described by the inclusion

$$\{z_{-l}, \ldots, z_{-1}, z_0, \bar{v}_t, \bar{\xi}_t\} \in M \tag{1.7}$$

The domain $Z_t(\bar{y}_t)$ consistent with the signal $\bar{y}_t$ realized is defined analogously. The determination of this domain's support function, which under analogous requirements on M is a convex set, reduces to solving the problem (1.4), (1.6), (1.2), (1.7). The dual problem has the form

$$\sup\Big\{\sum_{k=-l}^{-1}\langle\psi_{k+l+1}, Cz_k\rangle + \langle\psi_0, z_0\rangle + \sum_{k=0}^{t-1}(\langle\psi_{k+1}, v_k\rangle + \langle\eta_k, \xi_k\rangle - \langle\eta_k, y_k\rangle) : \{z_{-l}, \ldots, z_0, \bar{v}_t, \xi_t\} \in M\Big\} \to \inf \tag{1.8}$$

$$\psi_k = \psi_{k+1}A + \psi_{k+l+1}C + \eta_k B, \quad k = 0, 1, \ldots, t-1, \quad \psi_t = \psi, \quad \psi_{t+1} = \ldots = \psi_{t+l} = 0$$

The corresponding duality theorem is stated analogously.

Theorem 1.2. The value of the upper bound in problem (1.4), (1.6), (1.2), (1.7) and the lower bound in problem (1.8) coincide.

Below the forms of the functionals in (1.5) and (1.8) are defined more precisely for constraints (1.3) and (1.7) of concrete forms, while the dual problem is solved by the dynamic programming method. In this connection the Bellman function determines the support function of the domain consistent with the signal realized and the recurrence relations defining the Bellman function describe the dynamics of the information domains.

2. Consistent quadratic constraints. Let the indeterminacy in system (1.1)–(1.3) be constrained by the set

$$M = \Big\{\{z_0, \bar{v}_t, \bar{\xi}_t\} : \langle z_0, z_0\rangle + \sum_{k=0}^{t-1}(\langle v_k, v_k\rangle + \langle\xi_k, \xi_k\rangle) \leqslant 1\Big\} \tag{2.1}$$

(M is the unit ball in $R^{n(t+1)} \times R^{mt}$). In this case

$$\sup\Big\{\langle\psi_0, z_0\rangle + \sum_{k=0}^{t-1}(\langle\psi_{k+1}, v_k\rangle + \langle\eta_k, \xi_k\rangle) : \{z_0, \bar{v}_t, \bar{\xi}_t\} \in M\Big\} = \Big(\langle\psi_0, \psi_0\rangle + \sum_{k=0}^{t-1}(\langle\psi_{k+1}, \psi_{k+1}\rangle + \langle\eta_k, \eta_k\rangle)\Big)^{1/2}$$

If we introduce the additional variable (ζ_k), then we can write problem (1.5) as

$$(\langle\psi_0, \psi_0\rangle + \zeta_0)^{1/2} - \sum_{k=0}^{t-1}\langle\eta_k, y_k\rangle \to \inf, \quad \psi_k = \psi_{k+1}A + \eta_k B \tag{2.2}$$

$$\zeta_k = \zeta_{k+1} + \langle\psi_{k+1}, \psi_{k+1}\rangle + \langle\eta_k, \eta_k\rangle,\ k = 0, \ldots, t-1,\ \psi_t = \psi,\ \zeta_t = 0$$

The Bellman equation and the boundary condition for problem (2.2) are /6/

$$\Omega_k(\psi, \zeta) = \inf_\eta\{\Omega_{k-1}(\psi A + \eta B, \zeta + \langle\psi, \psi\rangle + \langle\eta, \eta\rangle) - \langle\eta, y_{k-1}\rangle\},\ \Omega_0(\psi, \zeta) = (\langle\psi, \psi\rangle + \zeta)^{1/2} \tag{2.3}$$

If no additional information on the indeterminate noise comes in during the process, then according to Theorem 1.1 the support function of domain $Z_t(\bar{y}_t)$ coincides with $\Omega_t(\psi, 0)$ and relations (2.3) define the dynamics in time of the information domains. The following statement is given without proof.

Lemma 2.1. Let there be given a scalar $a \geqslant 0$, vectors b and c, and a positive definite matrix R such that $1 - \langle b, R^{-1}b\rangle \geqslant 0$ and $a - \langle c, R^{-1}c\rangle \geqslant 0$. Then the equality

$$\inf_\eta\{(a + 2\langle c, \eta\rangle + \langle\eta, R\eta\rangle)^{1/2} - \langle\eta, b\rangle\} = \langle b, R^{-1}c\rangle + (1 - \langle b, R^{-1}b\rangle)^{1/2}(a - \langle c, R^{-1}c\rangle)^{1/2} \tag{2.4}$$

is valid. (If $1 - \langle b, R^{-1}b\rangle = 0$, then the lower bound in (2.4) is not achieved).

Theorem 2.1. If the indeterminate perturbations in (1.1) and (1.2) are specified by the consistent quadratic constraints (2.1), then the domain $Z_k(\bar{y}_k)$ is an ellipsoid with the support function

$$W_k(\psi \mid \bar{y}_k) = \langle z_k^\circ, \psi\rangle + \varepsilon_k\langle\psi, P_k\psi\rangle^{1/2},\ k = 0, \ldots, t \tag{2.5}$$

The ellipsoid's parameters satisfy the following difference equations with initial conditions:

$$z_k^\circ = Az_{k-1}^\circ + (y_{k-1} - Bz_{k-1}^\circ)\, R_{k-1}^{-1}BP_{k-1}A^* \tag{2.6}$$
$$P_k = I + AP_{k-1}A^* - AP_{k-1}B^*R_{k-1}^{-1}BP_{k-1}A^*$$
$$\varepsilon_k^2 = \varepsilon_{k-1}^2 - \langle (y_{k-1} - Bz_{k-1}^\circ),\ R_{k-1}^{-1}(y_{k-1} - Bz_{k-1}^\circ)\rangle$$
$$z_0^\circ = 0,\ P_0 = I,\ \varepsilon_0 = 1 \tag{2.7}$$

Here I is the unit matrix and $R_k = I + BP_kB^*$. (Matrix R_k is not singular since without loss of generality we can take the rows of matrix B to be linearly independent).

Proof. By induction we show that the function

$$\Omega_k(\psi, \zeta) = \langle z_k^\circ, \psi\rangle + \varepsilon_k(\zeta + \langle \psi, P_k\psi\rangle)^{1/2}$$

where z_k°, P_k and ε_k satisfy Eqs. (2.6), is a solution of problem (2.3). Indeed,

$$\Omega_0(\psi, \zeta) = \langle z_0^\circ, \psi\rangle + \varepsilon_0(\zeta + \langle \psi, P_0\psi\rangle)^{1/2} = (\zeta + \langle \psi, \psi\rangle)^{1/2}$$

Further, let

$$\Omega_{k-1}(\psi, \zeta) = \langle z_{k-1}^\circ, \psi\rangle + \varepsilon_{k-1}(\zeta + \langle \psi, P_{k-1}\psi\rangle)^{1/2}$$

Then

$$\Omega_k(\psi, \zeta) = \inf_\eta \{\langle z_{k-1}^\circ, \psi A + \eta B\rangle + \varepsilon_{k-1}(\zeta + \langle \psi, \psi\rangle +$$
$$\langle \eta, \eta\rangle + \langle (\psi A + \eta B),\ P_{k-1}(\psi A + \eta B)\rangle)^{1/2} - \langle \eta, y_{k-1}\rangle\} =$$
$$\langle Az_{k-1}^\circ, \psi\rangle + \varepsilon_{k-1}\inf_\eta \{(a + 2\langle c, \eta\rangle + \langle \eta, R_{k-1}\eta\rangle)^{1/2} - \langle \eta, b\rangle\}$$
$$a = \zeta + \langle \psi, (I + AP_{k-1}A^*)\psi\rangle,\ c = \psi AP_{k-1}B^*$$
$$R_{k-1} = I + BP_{k-1}B^*,\ b = \varepsilon_{k-1}^{-1}(y_{k-1} - Bz_{k-1}^\circ)$$

It can be shown that the hypotheses of Lemma 2.1 are satisfied for a, b, c and R_{k-1} defined thus, and, consequently,

$$\Omega_k(\psi, \zeta) = \langle Az_{k-1}^\circ, \psi\rangle + \varepsilon_{k-1}\langle b, R_{k-1}^{-1}c\rangle + \varepsilon_{k-1}(1 -$$
$$\langle b, R_{k-1}^{-1}b\rangle)^{1/2}(a - \langle c, R_{k-1}^{-1}c\rangle)^{1/2} = \langle z_k^\circ, \psi\rangle + \varepsilon_k(\zeta +$$
$$\langle \psi, P_k\psi\rangle)^{1/2}$$

where $z_k^\circ, P_k, \varepsilon_k$ satisfy relations (2.6).

We proceed to solve problem (1.8). We assume that $l = 1,\ z_{-1} = 0,\ \{z_0, \bar{v}_t, \bar{\xi}_t\} \in M$, M is determined from (2.1). Then

$$\sup\left\{\sum_{k=-l}^{-1}\langle \psi_{k+l+1}, Cz_k\rangle + \langle \psi_0, z_0\rangle + \sum_{k=0}^{t-1}(\langle \psi_{k+1}, v_k\rangle + \langle \eta_k, \xi_k\rangle) : z_{-1} = 0,\quad \{z_0, \bar{v}_t, \bar{\xi}_t\} \in M\right\} =$$
$$\left(\langle \psi_0, \psi_0\rangle + \sum_{k=0}^{t-1}(\langle \psi_{k+1}, \psi_{k+1}\rangle + \langle \eta_k, \eta_k\rangle)\right)^{1/2}$$

Introducing the additional scalar variable ζ_k, we arrive at a problem analogous to (2.2), where the equations of the adjoint system and the missing boundary conditions are taken from (1.8). We make use of Krasovskii's ideas /7/ to solve this problem by the dynamic programming method. If $\Omega_k(\psi, \chi, \zeta)$ is the Bellman function for this problem, then the dynamic programming equation takes the form

$$\Omega_{k+1}(\psi, \chi, \zeta) = \inf_\eta \{\Omega_k(\psi A + \eta B + \chi C, \psi, \zeta + \langle \psi, \psi\rangle + \langle \eta, \eta\rangle) - \langle \eta, y_k\rangle\},\ k = 0, 1, \ldots, t-1 \tag{2.8}$$

$$\Omega_0(\psi; \chi, \zeta) = (\zeta + \langle \psi, \psi\rangle)^{1/2}$$

Using Lemma 2.1, by induction it can be shown that the function

$$\Omega_k(\psi, \chi, \zeta) = \langle z_k^\circ, \psi\rangle + \langle x_k^\circ, \chi\rangle + \varepsilon_k(\zeta + \langle \psi, P_k\psi\rangle + \langle \chi, Q_k\chi\rangle + \langle \psi, S_k\chi\rangle + \langle \chi, S_k^*\psi\rangle)^{1/2},\ k = 0, 1, \ldots, t$$

is a solution of Eqs. (2.8) when $z_k^\circ, x_k^\circ, P_k, Q_k, S_k, \varepsilon_k$ satisfy the following realations:

$$z_{k+1}^\circ = Az_k^\circ + x_k^\circ + (y_k - Bz_k^\circ)R_k^{-1}B(S_k + P_kA^*) \tag{2.9}$$
$$x_{k+1}^\circ = Cz_k^\circ + (y_k - Bz_k^\circ)R_k^{-1}BP_kC^*$$
$$P_{k+1} = I + Q_k + AP_kA^* + AS_k + S_k^*A^* - (AP_k + S_k^*)\times B^*R_k^{-1}B(S_k + P_kA^*)$$

$$Q_{k+1} = CP_kC^* - CP_kB^*R_k^{-1}BP_kC^*$$
$$S_{k+1} = (AP_k + S_k^*)(I - B^*R_k^{-1}BP_k)C^*$$

with the initial conditions

$$z_0^\circ = x_0^\circ = 0,\ P_0 = I,\ Q_0 = 0,\ S_0 = 0,\ \varepsilon_0 = 1 \tag{2.10}$$

Here, as above, $R_k = I + BP_kB^*$ and ε_k is determined from the last relation in (2.6). If additional information on the perturbations' realizations does not come in during the process, then by virtue of the boundary conditions from (1.8) and (2.2)

$$W_k(\psi \mid \bar{y}_k) = \Omega_k(\psi, 0, 0) \tag{2.11}$$

Consequently, we have valid the following theorem.

Theorem 2.2. If the indeterminate perturbations in system (1.8), (1.2) are specified by the consistent quadratic constraints (2.1), then domain $Z_k(\bar{y}_k)$ is an ellipsoid with the support function

$$W_k(\psi \mid \bar{y}_k) = \langle z_k^\circ, \psi\rangle + \varepsilon_k \langle \psi, P_k\psi\rangle^{1/2}, \quad k = 0, 1, \ldots, t$$

and the ellipsoid's parameters satisfy Eqs.(2.9) and (2.10).

3. Geometric constraints. We now assume that the set M in (1.3) is defined as follows:

$$M = \{\{z_0, \bar{v}_t, \bar{\xi}_t\} : z_0 \in Z^0,\ v_k \in V,\ \xi_k \in \Xi,\ k = 0, \ldots, t-1\} \tag{3.1}$$

where Z°, V and Ξ are closed convex sets in R^n, R^n and R^m, respectively. In this case the information domains $Z_k(\bar{y}_k)$ do not possess as regular a structure as under the condition of consistent quadratic constraints on the perturbations, and it becomes difficult to obtain the filtering equations of form (2.6) and (2.7). Under such conditions it is convenient to have difference equations describing the dynamics of the information domains. With due regard to (3.1) we obtain

$$\sup\left\{\langle\psi_0, z_0\rangle + \sum_{k=0}^{t-1}(\langle\psi_{k+1}, v_k\rangle + \langle\eta_k, \xi_k\rangle) : \{z_0, \bar{v}_t, \bar{\xi}_t\} \in M\right\} = W(\psi_0 \mid Z^\circ) + \sum_{k=0}^{t-1}(W(\psi_{k+1} \mid V) + W(\eta_k \mid \Xi))$$

where $W(\cdot \mid Z^\circ)$, $W(\cdot \mid V)$ and $W(\cdot \mid \Xi)$ are the support functions of sets Z°, V and Ξ, respectively. Therefore, from the dynamic programming equations for problem (1.5) follows.

Theorem 3.1. Let $Z_k(\bar{y}_k)$ be the domain consistent with the signal $\bar{y}_k$ realized. Then the support function of set $Z_k(\bar{y}_k)$ satisfies the recurrence equation

$$W_{k+1}(\psi \mid \bar{y}_{k+1}) = \inf_\eta \{W_k(\psi A + \eta B \mid \bar{y}_k) + W(\eta \mid \Xi) - \langle\eta, y_k\rangle\} + W(\psi \mid V), \quad k = 0, \ldots, t-1 \tag{3.2}$$

$$W_0(\psi \mid \bar{y}_0) = W(\psi \mid Z^\circ) \tag{3.3}$$

This theorem has been proved in /8/ by some other methods.

Example 3.1. Let $V = \{0\}$, $\Xi = \{0\}$, $Z^\circ = R^n$. In this case Eq.(3.2) becomes

$$W_{k+1}(\psi \mid \bar{y}_{k+1}) = \inf_\eta \{W_k(\psi A + \eta B \mid \bar{y}_k) - \langle\eta, y_k\rangle\}$$

Hence, allowing for (3.3), we obtain that

$$W_t(\psi \mid \bar{y}_t) = \begin{cases} \langle\psi, A^t z_0\rangle, & \text{if } \exists \bar{\eta}_t : \psi A + \eta_0 B + \ldots + \eta_{t-1} BA^{t-1} = 0 \\ +\infty & \text{otherwise} \end{cases} \tag{3.4}$$

From (3.4) follows, in particular, the known observability criterion

$$\operatorname{Rank} \| B^*, A^*B^*, \ldots, A^{*n-1}B^* \| = n$$

(* denotes transposed matrices).

Example 3.2. Let $B = I$, $Z^\circ = R^n$, let matrix A be nonsingular and let the perturbation occur only in the measurement equations. Then

$$W_{k+1}(\psi \mid \bar{y}_{k+1}) = \inf_\eta \{W_k(\psi A + \eta \mid \bar{y}_k) + W(\eta \mid \Xi) - \langle\eta, y_k\rangle\}$$

Therefore, the estimate

$$W_k(\psi \mid \bar{y}_k) \leqslant \Lambda_k(\psi \mid \bar{y}_k) = \min_{1 \leqslant i \leqslant k} \{\langle A^i y_{k-i}, \psi\rangle + W(-\psi A^i \mid \Xi)\}$$

is valid. Furthermore, the equality

$$W_k(\psi \mid \bar{y}_k) = \Lambda_k^{**}(\psi \mid \bar{y}_k) \tag{3.5}$$

whose right-hand side is the second adjoint function /9/, holds.

Let us consider the nonlinear system

$$z_{k+1} = f(z_k, v_k),\ y_k = g(z_k, \xi_k),\ k = 0, \ldots, t-1 \tag{3.6}$$

As before, the perturbations $\{z_0, \bar{v}_t, \bar{\xi}_t\}$ satisfy the geometric constraints (3.1).

Theorem 3.2. Let $Z_k(\bar{y}_k)$ be the domain consistent with the signal realized in system (3.6). Then the relation

$$Z_k(\bar{y}_k) = f(Z_{k-1}(\bar{y}_{k-1}) \cap \Phi(y_{k-1}), V),\quad \Phi(y) = \{z : \exists \xi \in \Xi,\ g(z, \xi) = y\} \tag{3.7}$$

is valid.

The theorem's proof follows simply from the definition of $Z_k(\bar{y}_k)$. An analogous statement was given in /8/ for the case of linear systems. From Theorem 3.2 it follows that relation (3.2) can be treated as the application of the infimal convolution operation /9/ to the support functions of the corresponding sets for linear $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$. Thus, relation (3.7) opens up certain possibilities for the analysis of information domains in the case of nonlinear systems. Assume that function $f(z, v)$ is linear, set $\Phi(y)$ is closed and convex for any admissible y, and Z^0 is bounded. Let $W_k(\cdot \mid \bar{y}_k)$, $k = 0, \ldots, t$ and $W(\cdot \mid \Phi(y))$ be the support functions of sets $Z_k(\bar{y}_k)$ and $\Phi(y)$, respectively. Then from (3.7) follows

Theorem 3.3. The support functions of domains $Z_k(\bar{y}_k)$ consistent with signal $\bar{y}_k$ in (1.1), (3.6), (3.1) satisfy the recurrence equation

$$W_{k+1}(\psi \mid \bar{y}_{k+1}) = W(\psi \mid V) + \inf_{\chi} \{W_k(\psi A + \chi \mid \bar{y}_k) + W(-\chi \mid \Phi(y_k))\} \tag{3.8}$$

with initial condition (3.3).

If the assumptions in Example 3.2 are fulfilled, then

$$W_k(\psi \mid \bar{y}_k) \leqslant \Lambda_k(\psi \mid \bar{y}_k) = \min\{W(\psi A^{k-i} \mid \Phi(y_i)) : 0 \leqslant i \leqslant k-1\}$$

It can be shown that analogously to (3.5)

$$W_k(\psi \mid \bar{y}_k) = \Lambda_k^{**}(\psi \mid \bar{y}_k)$$

(The requirement that matrix A be nonsingular can be eliminated by examining the corresponding functions on the subspace generated by the linearly-independent rows of A).

Example 3.3. Let $g(z, \xi) = z + |z|\xi$, $|\xi| \leqslant 1$. Then $\Phi(y) = \{z : \langle z, y\rangle \geqslant {}^1/_2 |y|^2\}$ and

$$W(\psi \mid \Phi(y)) = \begin{cases} {}^1/_2\alpha |y|^2, & \text{if} \quad \psi = \alpha y,\ \alpha \geqslant 0 \\ +\infty & \text{otherwise} \end{cases}$$

Consequently, Eq.(3.8) can be written as

$$W_k(\psi \mid \bar{y}_k) = W(\psi \mid V) + \inf_{\alpha \geqslant 0} \{W_{k-1}(\psi A + \alpha y_{k-1} \mid \bar{y}_{k-1}) - {}^1/_2\, \alpha\, |y_{k-1}|^2\}$$

The task of determining $W_k(\psi \mid \bar{y}_k)$ is reduced in this case to solving k one-dimensional optimization problems.

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